
Tutorial Sheet-6: Group Theory

- (1) Give two reasons why the set of odd integers under addition is not a group.
- (2) Show that $\{1, 2, 3\}$ under multiplication modulo 4 is not a group but that $\{1, 2, 3, 4\}$ under multiplication modulo 5 is a group.
- (3) Prove that in a group, $(a^{-1})^{-1} = a$ for all a .
- (4) Prove that a group G is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a and b in G .
- (5) Let p be a prime number and G be a group such that $|G| = p$. The group G is cyclic.
- (6) Is D_3 (the set of symmetries of an equilateral triangle) Abelian?
- (7) Prove that a group G is Abelian if and only if $(ab)^2 = a^2b^2$ for all a and b in G .
- (8) Prove that an Abelian group with two elements of order 2 must have a subgroup of order 4.
- (9) Let $\Phi : G \rightarrow H$ be a homomorphism. Show that $\Phi(e_G) = e_H$ and $\Phi(a^{-1}) = (\Phi(a))^{-1}$.
- (10) Let G be a group such that $|G| \leq 5$. The group G is abelian.
- (11) Write order of elements of \mathbb{Z}_n under addition modulo n .
- (12) Show that the set $Z[x]$ of all polynomials in the variable x with integer coefficients under ordinary addition and multiplication is a commutative ring with unity $f(x) = 1$.
- (13) Show that $2\mathbb{Z} \cup 3\mathbb{Z}$ is not a subring of \mathbb{Z} .
- (14) The ring $\{0, 2, 4, 6, 8\}$ under addition and multiplication modulo 10 has a unity (multiplicative identity). Find it.
- (15) Let R be a ring with unity 1. Show that $S = \{n \cdot 1 : n \in \mathbb{Z}\}$ is a subring of R .
- (16) Suppose that a belongs to a ring and $a^4 = a^2$. Prove that $a^{2n} = a^2$ for all $n \geq 1$.
- (17) Let F be a finite field with n elements. Prove that $x^{n-1} = 1$ for all nonzero x in F , 1 denotes the multiplicative identity of F .